

4.5 (Standard) Amplitude modulation: AM

4.60. DSB-SC amplitude modulation (which is summarized in Figure 26) is easy to understand and analyze in both time and frequency domains. However, analytical simplicity is not always accompanied by an equivalent simplicity in practical implementation.

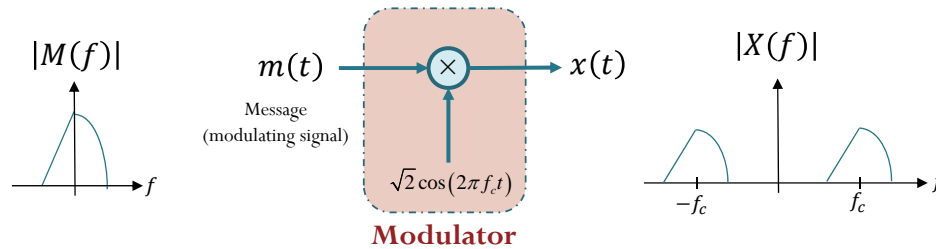


Figure 26: DSB-SC modulation.

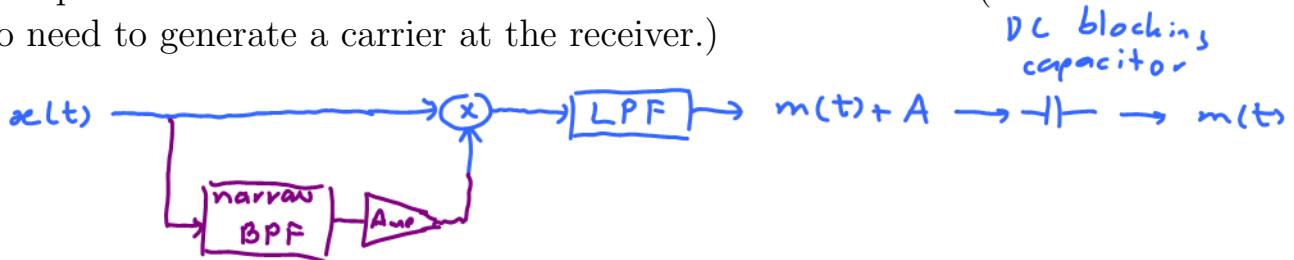
Problem: The (coherent) demodulation of DSB-SC signal requires the receiver to possess a carrier signal that is synchronized with the incoming carrier. This requirement is not easy to achieve in practice because the modulated signal may have traveled hundreds of miles and could even suffer from some unknown frequency shift.

4.61. If a carrier component is transmitted along with the DSB signal, demodulation can be simplified.

$$x(t) = m(t)\cos(2\pi(f_c)t) + A\cos(2\pi(f_c)t)$$

$$= (m(t) + A)\cos(2\pi(f_c)t)$$

- (a) The received carrier component can be extracted using a narrowband bandpass filter and can be used as the demodulation carrier. (There is no need to generate a carrier at the receiver.)



- (b) If the carrier amplitude is sufficiently large, the need to generate a demodulation carrier can be avoided completely.

- This will be the focus of this section.

$$\frac{A}{2} \delta(f - f_c) + \frac{A}{2} \delta(f - (-f_c)) + \frac{1}{2} M(f - f_c) + \frac{1}{2} M(f - (-f_c))$$

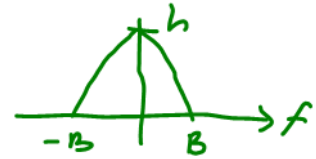
Definition 4.62. For AM, the transmitted signal is typically defined as

$$x_{AM}(t) = \underbrace{(A + m(t)) \cos(2\pi f_c t)}_{A(t)} = \underbrace{A \cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{m(t) \cos(2\pi f_c t)}_{\text{sidebands}}$$

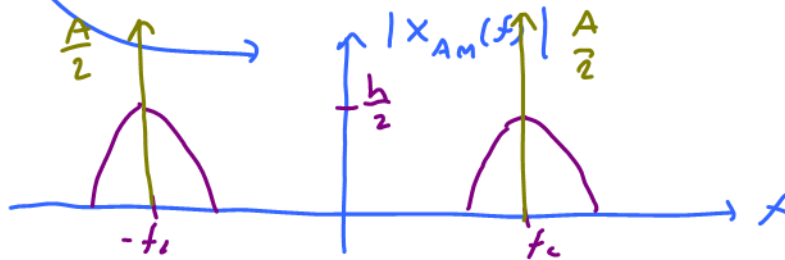
Assumptions for $m(t)$:

(a) **Band-limited to B** : that is, $|M(f)| = 0$ for $|f| > B$.

(b) **Bounded between $-m_p$ and m_p** , that is, $|m(t)| \leq m_p$.



4.63. Spectrum of $x_{AM}(t)$:



- Basically the same as that of DSB-SC signal except for the two additional impulses (**discrete** spectral component) at the carrier frequency $\pm f_c$.

in the earlier section

- This is why we say the DSB-SC system is a **suppressed carrier** system.

Definition 4.64. Consider a signal $A(t) \cos(2\pi f_c t)$. If $A(t)$ varies slowly in comparison with the sinusoidal carrier $\cos(2\pi f_c t)$, then the **envelope** $E(t)$ of $A(t) \cos(2\pi f_c t)$ is $|A(t)|$.

4.65. **Envelope of AM signal:** For AM signal, $A(t) \equiv A + m(t)$ and

$$E(t) = |A + m(t)|.$$

See Figure 27.

Case (a) If $\forall t, A(t) > 0$, then $E(t) = A(t) = A + m(t)$

- The envelope has the same shape as $m(t)$.
- Enable envelope detection: Extract $m(t)$ from the envelope.

Case (b) If $\exists t, A(t) < 0$, then $E(t) \neq A(t)$.

- The envelope shape differs from the shape of $m(t)$ because the negative part of $A + m(t)$ is rectified.
 - This is referred to as **phase reversal** and envelope distortion.

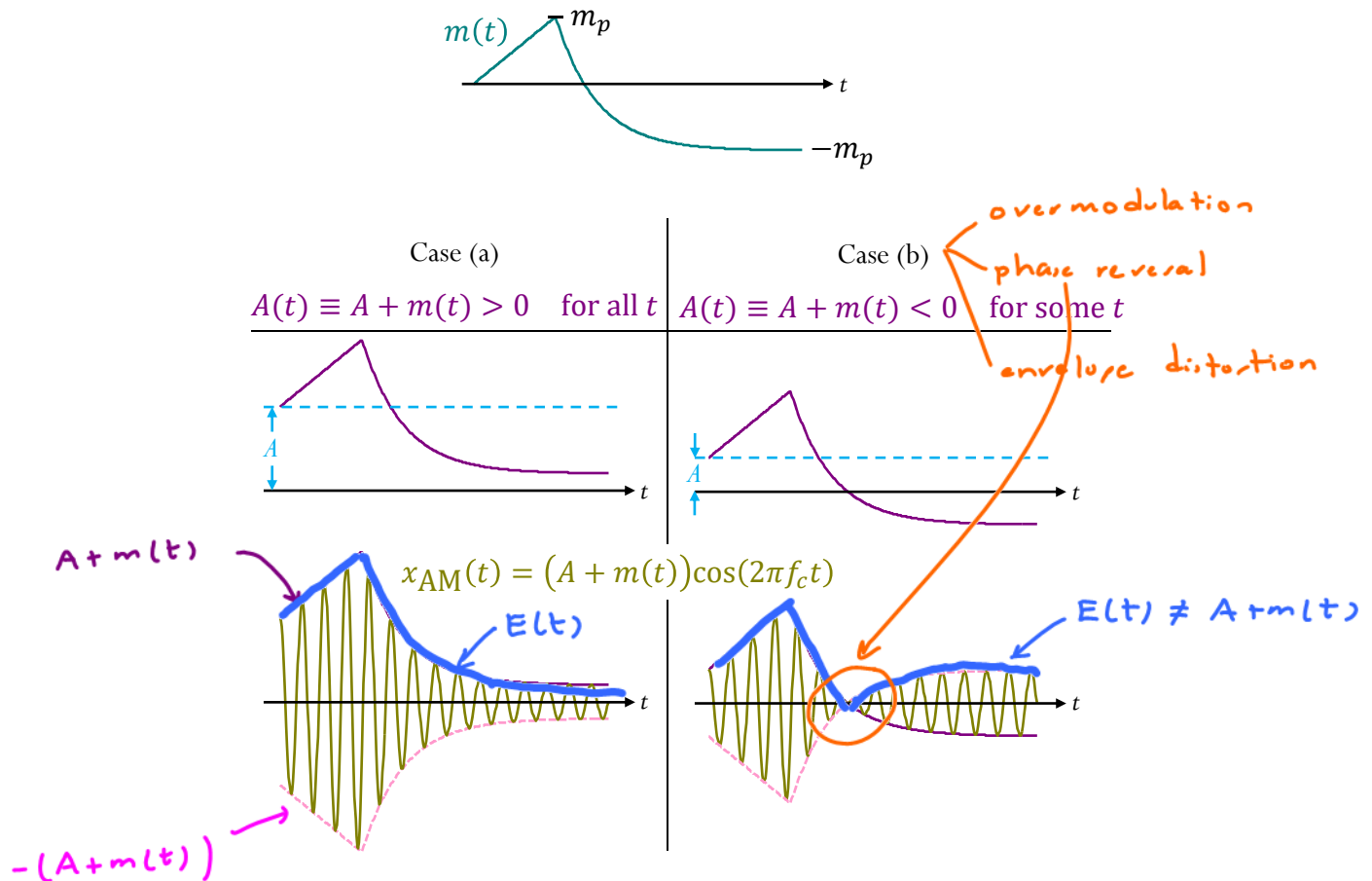


Figure 27: AM signal and its envelope [6, Fig 4.8]

Definition 4.66. The positive constant

$$\mu \equiv \frac{\max_t (\text{envelope of the sidebands})}{\max_t (\text{envelope of the carrier})} = \frac{\max_t |m(t)|}{\max_t |A|} = \frac{m_p}{A}$$

Assumptions: $A, m_p > 0$

is called the **modulation index**.

- The quantity $\mu \times 100\%$ is often referred to as the **percent modulation**.

$$\mu \leq 1 \quad (100\%)$$

$$m_p \leq A \Rightarrow -m_p \geq -A$$

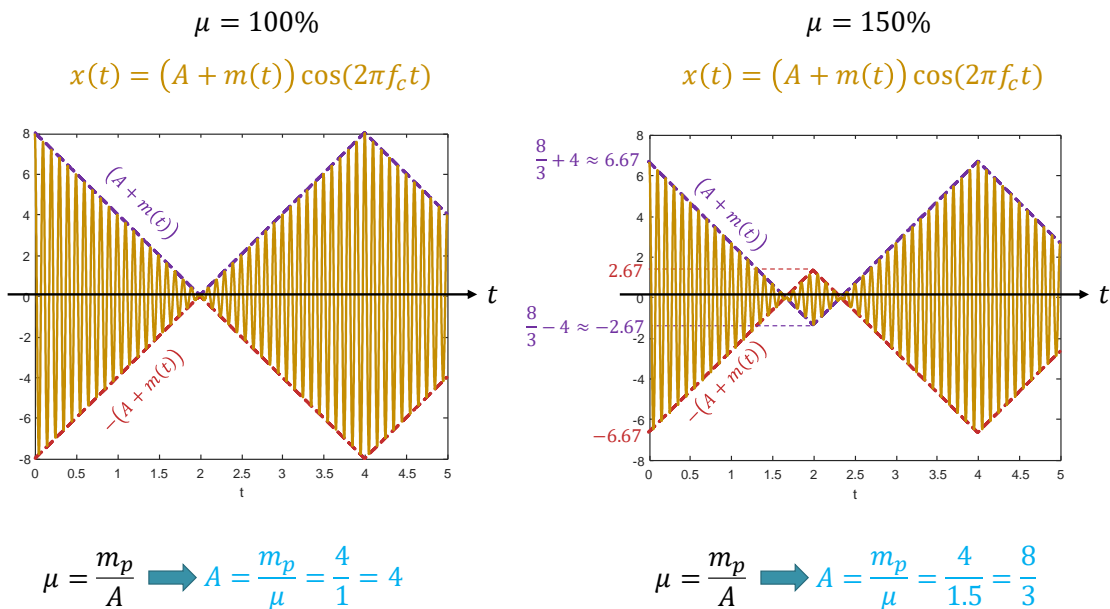
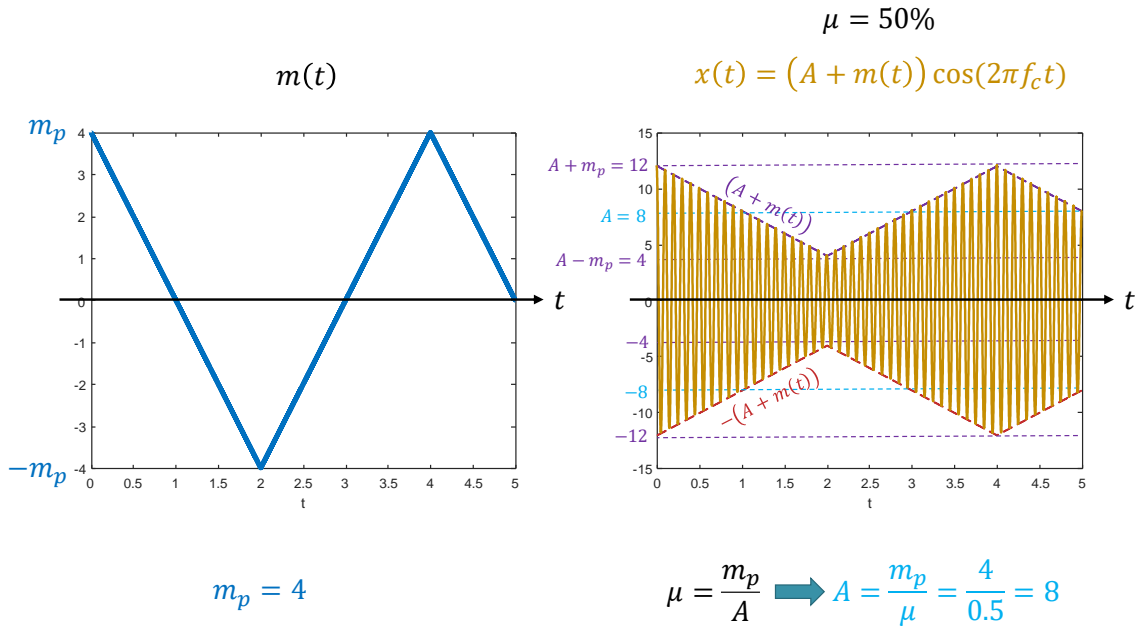
$$A + m(t) \geq A + (-m_p) \geq A + (-A) = 0$$

\Rightarrow case (a)

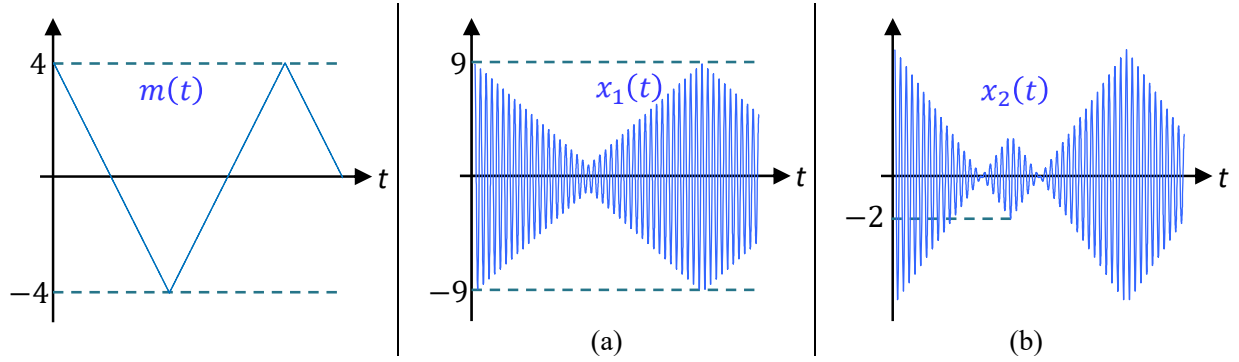
overmodulation $\mu > 1$

$A + m(t)$ may become < 0 for some t
 \Rightarrow case (b)

Example 4.67. Suppose $m(t)$ is plotted below. Assume that the carrier frequency f_c is large enough. Plot the corresponding AM signal $x_{AM}(t)$ when the modulation index is (a) 50%, (b) 100%, and (c) 150%.

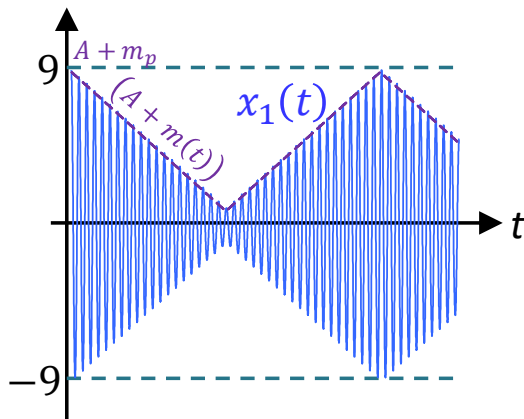


Example 4.68. Consider an AM transmission of the message $m(t)$ shown below. Two corresponding AM signals are plotted. Determine the modulation index used in each signal.



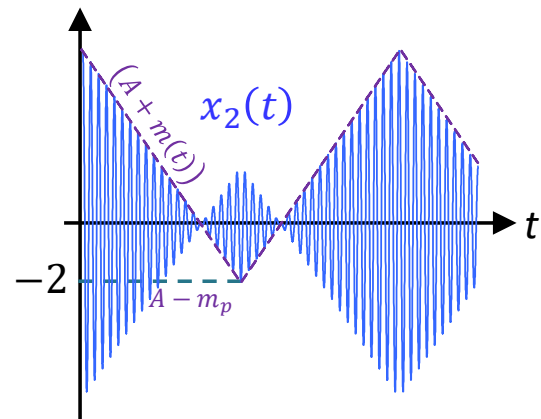
Solution:

(a)



$$\begin{aligned} A + m_p &= 9 \\ A + 4 &= 9 \\ A &= 5 \\ \mu &= \frac{m_p}{A} = \frac{4}{5} = 0.8 = 80\% \end{aligned}$$

(b)



$$\begin{aligned} A - m_p &= -2 \\ A - 4 &= -2 \\ A &= 2 \\ \mu &= \frac{m_p}{A} = \frac{4}{2} = 2 = 200\% \end{aligned}$$

Example 4.69. Consider a sinusoidal (pure-tone) message $m(t) = A_m \cos(2\pi f_m t)$. Suppose $A = 1$. Then, $\mu = A_m$. Figure 28 shows the effect of changing the modulation index on the modulated signal.

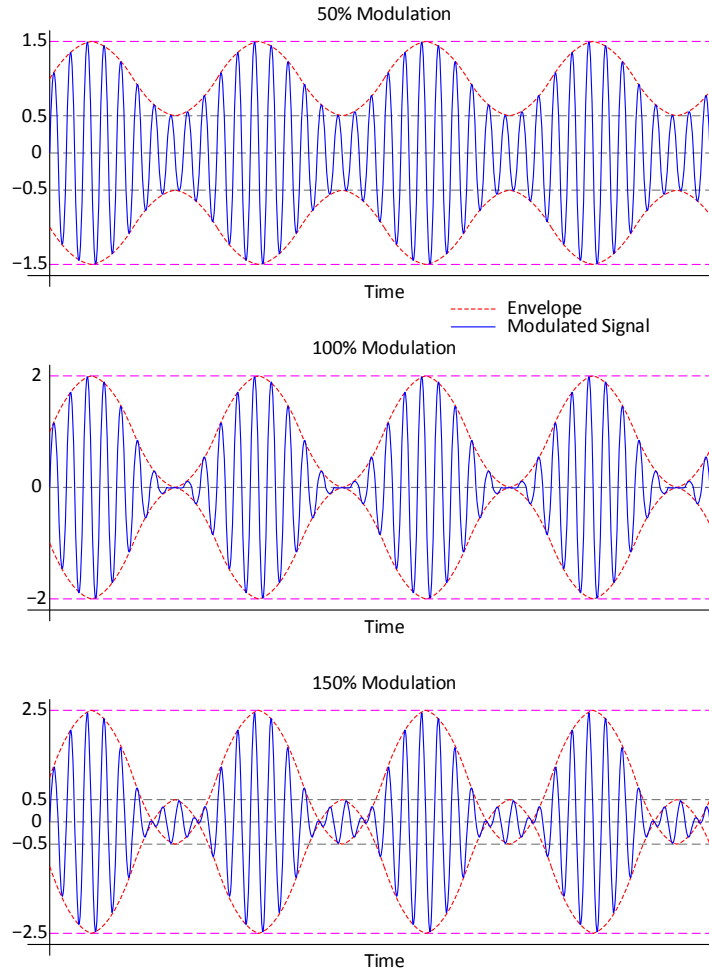


Figure 28: Modulated signal in standard AM with sinusoidal message

4.70. It should be noted that the ratio that defines the modulation index compares the maximum of the two envelopes. In other references, the notation for the AM signal may be different but the idea (and the corresponding motivation) that defines the modulation index remains the same.

- In the textbook by Carlson and Crilly, [3, p 163], it is assumed that $m(t)$ is already scaled or normalized to have a magnitude not exceeding unity ($|m(t)| \leq 1$) [3, p 163]. There,

$$x_{\text{AM}}(t) = A_c(1 + \mu m(t)) \cos(2\pi f_c t) = \underbrace{A_c \cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{A_c \mu m(t) \cos(2\pi f_c t)}_{\text{sidebands}}.$$

- $m_p = 1$
- The modulation index is then

$$\frac{\max_t (\text{envelope of the sidebands})}{\max_t (\text{envelope of the carrier})} = \frac{\max_t |A_c \mu m(t)|}{\max_t |A_c|} = \frac{|A_c \mu|}{|A_c|} = \mu.$$

- In [15, p 116],

$$x_{\text{AM}}(t) = A_c \left(1 + \mu \frac{m(t)}{m_p} \right) \cos(2\pi f_c t) = \underbrace{A_c \cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{A_c \mu \frac{m(t)}{m_p} \cos(2\pi f_c t)}_{\text{sidebands}}.$$

- The modulation index is then

$$\frac{\max_t (\text{envelope of the sidebands})}{\max_t (\text{envelope of the carrier})} = \frac{\max_t \left| A_c \mu \frac{m(t)}{m_p} \right|}{\max_t |A_c|} = \frac{|A_c| \mu \frac{m_p}{m_p}}{|A_c|} = \mu.$$

4.71. Power of the transmitted signals.

- (a) In DSB-SC system, recall, from 4.40, that, when

$$x(t) = m(t) \cos(2\pi f_c t)$$

with f_c sufficiently large, we have

$$P_x = \frac{1}{2} P_m.$$

All transmitted power are in the sidebands which contain message information.

- (b) In AM system,

$$x_{\text{AM}}(t) = \underbrace{A \cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{m(t) \cos(2\pi f_c t)}_{\text{sidebands}}.$$

If we assume that the average of $m(t)$ is 0 (no DC component), then the spectrum of the sidebands $m(t) \cos(2\pi f_c t + \theta)$ and the carrier $A \cos(2\pi f_c t + \theta)$ are non-overlapping in the frequency domain. Hence, when f_c is sufficiently large

$$P_x = \frac{1}{2} A^2 + \frac{1}{2} P_m.$$




- Efficiency:

$$\text{Eff.} = \frac{\frac{P_m}{2}}{\frac{A^2}{2} + \frac{P_m}{2}} = \frac{P_m}{A^2 + P_m} = \frac{1}{\frac{A^2}{P_m} + 1} = \frac{1}{1 + \frac{m_p^2}{\mu^2 P_m}} = \frac{1}{1 + \frac{1}{\mu^2}}$$

$\mu = \frac{m_p}{A} \Rightarrow A = \frac{m_p}{\mu}$

if $m(t)$ is sinusoidal, best = 1, $P_m = \frac{m_p^2}{2}$

Ex. 

- For high power efficiency, we want small $\frac{m_p^2}{\mu^2 P_m}$.

- By definition, $|m(t)| \leq m_p$. Therefore, $\frac{m_p^2}{P_m} \geq 1$.

$$|m(t)|^2 \leq m_p^2 \Rightarrow P_m = \langle |m(t)|^2 \rangle \leq m_p^2$$

- Want μ to be large. However, when $\mu > 1$, we have phase reversal. So, the largest value of μ is 1.

- The best power efficiency we can achieved is then 50%.

$$\frac{1}{1 + \frac{(1)}{(1)}} = \frac{1}{2} = 50\%$$

- Conclusion: at least 50% (and often close to 2/3 [3, p. 176]) of the total transmitted power resides in the carrier part which is independent of $m(t)$ and thus conveys no message information.

Example 4.72. Continue from Example 4.68. Suppose $m(t)$ is a periodic triangular wave with average power $\langle m^2(t) \rangle = \frac{16}{3}$. Calculate the corresponding value of the power efficiency for each case.

Solution:

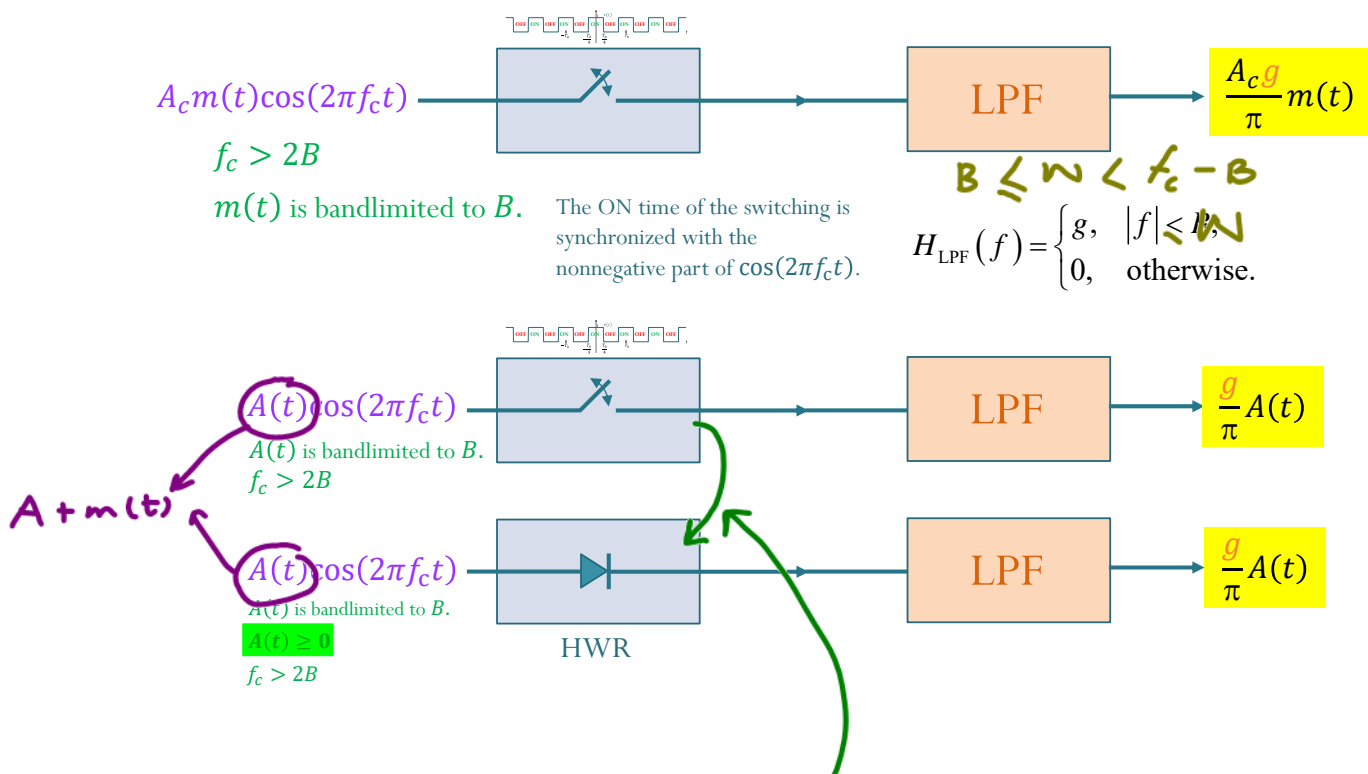
$x_{AM}(t)$	A	μ	Power Efficiency
$x_1(t)$	5	80%	$\text{Eff} = \frac{\frac{P_m}{2}}{\frac{A^2}{2} + \frac{P_m}{2}} = \frac{1}{\frac{A^2}{P_m} + 1} = \frac{1}{\frac{5^2}{16/3} + 1} = \frac{16}{91} \approx 0.1758 = 17.58\%$
$x_2(t)$	2	200%	$\text{Eff} = \frac{\frac{P_m}{2}}{\frac{A^2}{2} + \frac{P_m}{2}} = \frac{1}{\frac{A^2}{P_m} + 1} = \frac{1}{\frac{2^2}{16/3} + 1} = \frac{4}{7} \approx 0.5714 = 57.14\%$

4.73. From switching demodulator to rectifier demodulator:

- An AM signal can be demodulated using the same coherent demodulation technique that was used for DSB. However, the use of coherent demodulation negates the advantage of AM.
- Note that, conceptually, the received AM signal is the same as DSB-SC signal except that the $m(t)$ in the DSB-SC signal is replaced by $A(t) = A + m(t)$. We also assume that A is large enough so that $A(t) \geq 0$.
- The key equation of **switching demodulator** (60) is reexpressed here as:

$$\text{LPF}\{A(t) \cos(2\pi f_c t) \times 1[\cos(2\pi f_c t) \geq 0]\} = \frac{g}{\pi} A(t) \quad (61)$$

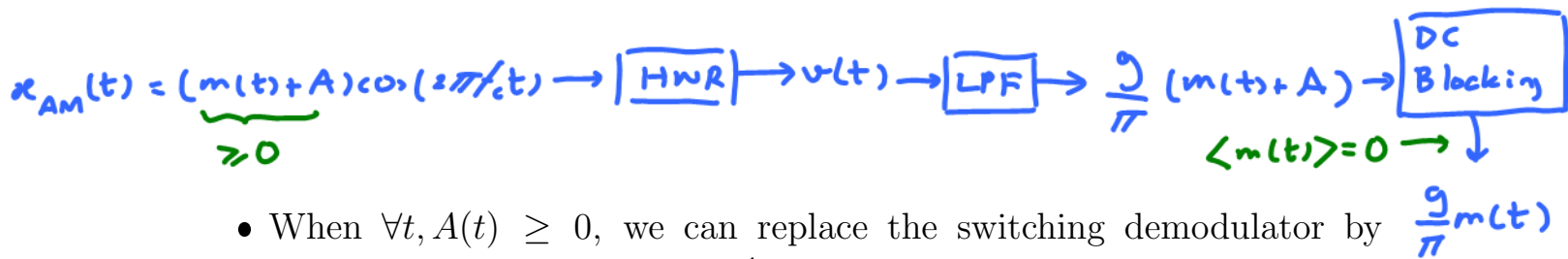
We noted before that this technique requires the switching to be in sync with the incoming cosine.



- The additional assumption that $A(t) \geq 0$ makes it possible to replace the switching demodulator with a half-wave rectifier.

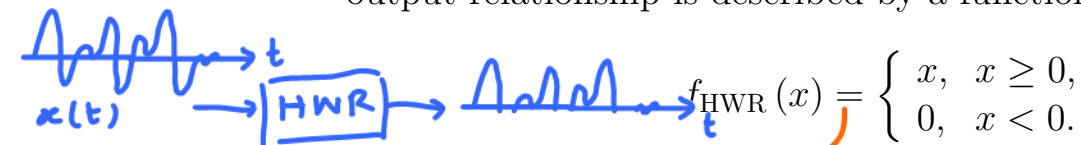
4.74. Demodulation of AM Signals via rectifier demodulator/detector:

The receiver will first recover $A + m(t)$ and then remove A .



- When $\forall t, A(t) \geq 0$, we can replace the switching demodulator by the **rectifier demodulator/detector**. In which case, we suppress the negative part of $y(t) = x_{AM}(t)$ using a diode (half-wave rectifier: HWR).

- Here, we define a HWR to be a memoryless device whose input-output relationship is described by a function $f_{HWR}(\cdot)$:



- Surprisingly, this is mathematically equivalent to a switching demodulator in (60) and (61).

$$v(t) = \begin{cases} x_{AM}(t), & x_{AM}(t) \geq 0, \\ 0, & x_{AM}(t) < 0 \end{cases} = \begin{cases} x_{AM}(t), & (A+m(t))\cos(2\pi f_c t) \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$= \begin{cases} x_{AM}(t), & \cos(2\pi f_c t) \geq 0, \\ 0, & \text{otherwise.} \end{cases} = x_{AM}(t) \times \begin{cases} 1, & \cos(2\pi f_c t) \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

Handwritten notes on the equations:

- Under $x_{AM}(t) \geq 0$: ≥ 0
- Under $(A+m(t))\cos(2\pi f_c t) \geq 0$: ≥ 0
- Under $\cos(2\pi f_c t) \geq 0$: $\rightarrow \text{rect in sec. 4.4}$

- It is in effect synchronous detection performed without using a local carrier [5, p 167].
- This method needs $A(t) \geq 0$ so that the sign of $A(t) \cos(2\pi f_c t)$ will be the same as the sign of $\cos(2\pi f_c t)$.
- The dc term $\frac{9A}{\pi}$ may be blocked by a capacitor.

$$g(t) \rightarrow \boxed{\text{DC Blocking}} \rightarrow g(t) - \langle g(t) \rangle$$

Handwritten notes on the equation:

- Under $\langle g(t) \rangle$: \uparrow 81
- Below the equation: **time average operator**

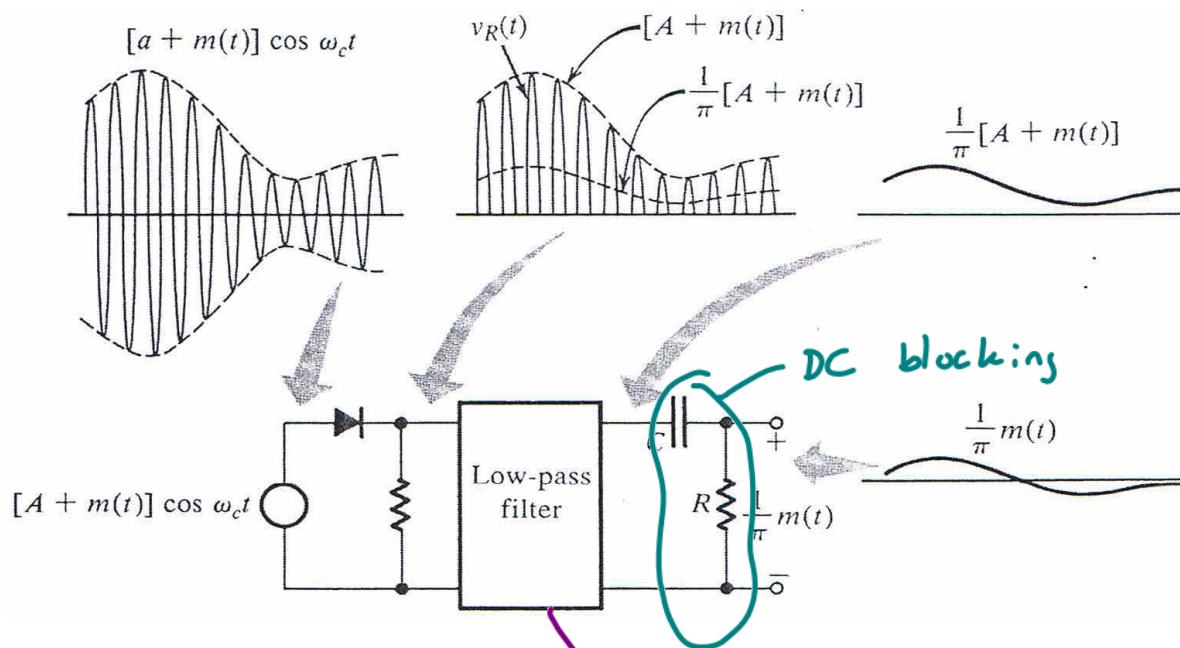


Figure 29: Rectifier detector for AM [6, Fig. 4.10]. Here, the gain in the passband of the LPF is set to $g = 1$

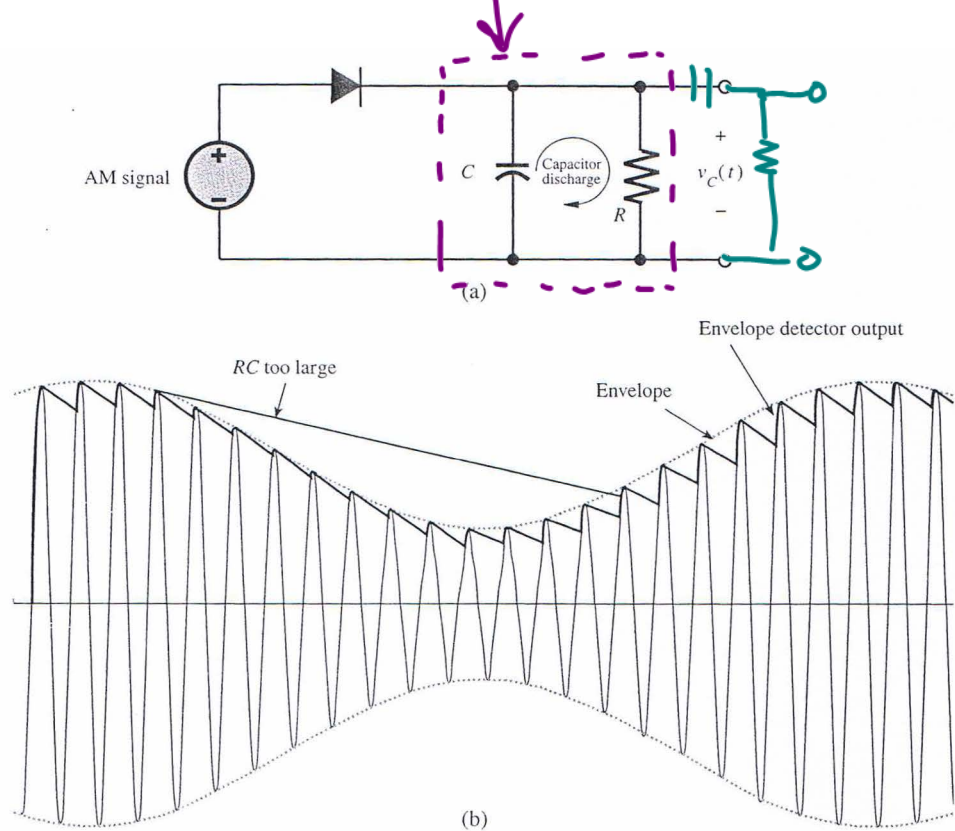


Figure 30: Envelope detector for AM [6, Fig. 4.11].

4.75. Demodulation of AM signal via *envelope detector*:

- Design criterion of RC:

$$2\pi B \ll \frac{1}{RC} \ll 2\pi f_c.$$

- The envelope detector output is $A + m(t)$ with a ripple of frequency f_c .
- The dc term can be blocked out by a capacitor or a simple RC high-pass filter.
- The ripple may be reduced further by another (low-pass) RC filter.

4.76. AM Trade-offs:

(a) *Disadvantages*:

- Higher **power** and hence **higher cost** required at the transmitter
- The carrier component is wasted power as far as information transfer is concerned.
- Bad for power-limited applications.

(b) *Advantages*:

- Coherent reference is not needed for demodulation.
- **Demodulator** (receiver) becomes simple and **inexpensive**.
- For **broadcast system** such as commercial radio (with a **huge number of receivers** for each transmitter),
 - any cost saving at the receiver is multiplied by the number of receiver units.
 - it is more economical to have one expensive high-power transmitter and simpler, less expensive receivers.

(c) Conclusion: Broadcasting systems tend to favor the trade-off by migrating cost from the (many) receivers to the (fewer) transmitters.

4.77. References: [3, p 198–199], [6, Section 4.3] and [14, Section 3.1.2].